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ON THE DEVELOPMENT OF THE TEMPERATURE CONCEPT

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Abstract

A brief outline of the history of development of the temperature concept in physics is given. Simultaneously, some persisting imperfections in the conceptual basis of classical thermodynamics closely related to the first and the second law of thermodynamics are discussed.

Keywords: second law of thermodynamics, temperature, thermometry

The concept of temperature, due to its practical significance in meteorology, medicine and technologies is one of the most commonly used physical concepts. In a civilised world, even small children (5–7 years old) are well acquainted with various types of thermometers giving the ‘temperature’ of sick children (!), of the out-door environment, or of the car engine. It should be noticed, however, that the medical thermometer is deemed by children to be rather a healing instrument, decisive for an imperative command to stay in bed, while the out-door thermometer decides how one has to be dressed, and the position of a pointer on the dial in the car thermometer has some importance for the well-being of the engine. There is, as a rule, no clear connection among these different kinds of ‘temperature’ given by particular instruments. For teenagers it is quite clear that all the things in the world have to be measured and compared, so that it is natural that an instrument called a ‘thermometer’ was devised for the determination of the ‘exact’ temperature – a quantity having something to do with our imperfect feeling of hotness and coldness. Invention of temperature is nothing but a further improvement of our modern life-style in comparison with that of our ancestors. Eventually, all adults believe that they know what temperature is. The only persisting problem is represented by various temperature scales and degrees, i.e. Fahrenheit, centigrade or Kelvin. The reason for their coexistence remains obscure and the common perception is that some of these degrees are probably more accurate, or simply better – in close analogy with dollars and other currencies.

In modern physics, temperature usually pretends to be a well-defined concept, intelligible for all and if not so, at least for experts. For instance, even such a critic

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and sceptic par excellence as *R*. Truesdell, founder of rational thermodynamics, explains this term by writing [1] that 'The body is at each time assigned a real number called temperature. This number is a measure of how hot the body is.' – a definition which is not too far from a plain tautology. On the other hand, more sophisticated definitions of temperature, based on statistical physics [2] or an axiomatic approach to phenomenology [3], are rather difficult to understand and/or apply to non-trivial experimental situations (involving e.g. quantum interference).

Modern thermal physics started to develop in the 17th century with the invention of the thermometer enabling quantitative studies of thermal phenomena to be made. This statement should not, however, be interpreted as that there was no scientific theory dealing with heat effects before this date. Equally wrong, is the widely spread opinion that, after the thermometer became a popular instrument, the then scholars had a clear idea of what temperature is and, by making experiments with thermometers, were aware of what they were actually doing.

It may be quite surprising that a very essential part of ancient natural philosophy consisted just of what we now call thermal physics, and that the theories and hypotheses worked out by these philosophers were more than one and a half centuries after the invention of thermometer still active. How was it possible to build up the predicative theory of thermal phenomena ignoring such a quantity as temperature? To give an answer to this question, it is worth saying a few words about these strange theories.

The forms of energy (in contemporary terminology) generally known to ancient people were only two, namely mechanical and thermal. (The knowledge of electrical energy, documented e.g. in the Bible should be considered as an exception.) From the corresponding physical disciplines, however, only mechanics and optics were accessible to mathematical description. The rest, dealing with the structure of matter and including thermal, meteorological, chemical or physiological phenomena, was treated only by means of verbal arguments and logical constructions.

The most representative theory of this type, formulated by Aristotle in the 4th century B. C., is based on the famous doctrine of four Elements [4]. According to this theory all objects in the nature are created of four Elements called water, earth, fire and air, by means of the action of four Qualities, namely coldness, dryness, hotness and humidity. Every body thus consists of passive Matter and active Form, the Matter being a proper mixture of elements and the Form a mixture of the said Qualities. Every Element tends to its natural place in the world and permanently possesses two Qualities, one of which is active (coldness, hotness) and the other passive (dryness, humidity) and one of which in dominant (primary Q) and the other submissive (secondary Q). Due to the enormous vastness of these relationships, graphical representation became very popular (Fig. 1) and later, it was even believed that formal manipulation with graphical symbols could be helpful for the solution of particular problems (cf. however, the modern theory of graphs). The hypothetical structure of matter, based on such a scheme, brings about an important consequence – the potential or intrinsic 'thermal' property of all existing substances. Thus, e.g. alcohol, gun-powder and pepper are intrinsically hot substances, active with respect to other bodies, while opium and snow are examples of intrinsically cold materials. Moreover, the antago-

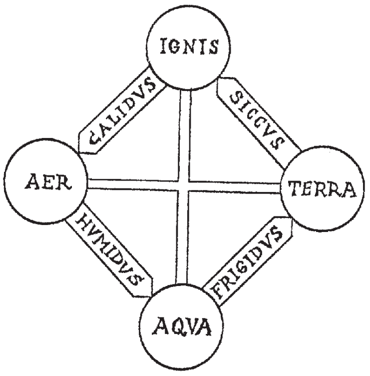

Fig. 1 Partially oriented graph illustrating various relationships of four Element theory

nistic nature (so called *contraria*) of different Elements and Qualities ensures eternal changes and movements of all things in the universe, in close analogy with well-known effects due to the combination love–hate in human society. These changes are, however, not completely free, but are submitted to the remarkable principle of *antiperistasis* controlling the relationship between two active Qualities – coldness and hotness. The principle can be formulated as follows [5]: The properties of any body which are bound up with coldness (hotness) tend to increase where the body is surrounded by a hot (cold) environment. This principle is akin to the more modern Le Chatelier–Braun principle which provides, in a lot of cases, correct qualitative predictions as concerns the direction of thermal processes. A typical example consistent with the principle of *antiperistasis* originates from Oinipides of Chios (5th century B.C.). 'Water in a deep well shows in winter the smallest degree of coldness, while in very hot days it is extraordinarily cold.' Interestingly, this statement is actually valid and is not only a consequence of our subjective feelings, but has been confirmed by careful hydrological studies [6]. There are numerous successful applications of the principle of *antiperistasis*, but there are also cases where it completely failed. (The same is, however, valid for the Le Chatelier–Braun principle! [7]) For example, the dissolution of black gun-powder containing saltpetre led, contrary to expectation, not to the warming up but to cooling. Such exceptions were either neglected or used for discussion of other weak points of the doctrine. The most important problem, crucial for the theory, was the so-called problem of *primum frigidum*. While there was no doubt in which Element the hotness dwells – of course in fire – the primary seat of the coldness remained uncertain. This made the conclusions of the theory not very plausible. The problem of *primum frigidum* was never solved and disappeared only with the whole theory.

In spite of the fact, as we have seen, that the concept of temperature was superfluous for the general description of natural processes within the framework of Aristotle's theory, the term *temperatura* was frequently used by ancient physicians well before Avicenna (11th century A. D.) [5]. Their temperature was in close connection with the individual temperament and was given by a certain mixture of four Qualities

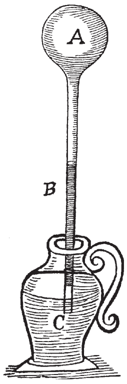

Fig. 2 A form of glass thermoscope used in the 17th century

which was necessary to maintain the Form of the tissues of the human body in a proper healthy state – *homeostasis*. But, in fact, these ancient physicians did not know how to determine this evidently crucial parameter. Probably the first attempt to define the state of the human body by objective physical measurements came from a group of Italian scientists at the beginning of the 17th century. Sanctorius (Santorio) studied experimentally the forces exerted by muscles, the content of various fluids in the body and the frequency of pulses using a *pulsologium* – an apparatus devised by Galileo. He tried, also, to measure an instantaneous characteristic of temperament, i.e. temperature, by means of a modified version of a very old device called a thermoscope, which had already been described by Philon of Byzantine (3rd century B. C.) and Heron of Alexandria (1st century A. D.) [8]. A later form of this instructive instrument is depicted in Fig. 2. It consists of a large bulb, A, hermetically attached to a thin glass tube, B, the end of which is immersed into water in the vessel, C. To prepare the apparatus for experiments, a few bubbles are driven out of the tube by a slight heating of the bulb A. After that, the instrument works as a gas dilatometer, sensitive to changes of temperature (but also to changes of the external pressure). The addition of a regular scale, made of glass pearls, to the pipe of the thermoscope enabled Sanctorius to judge the degree of the patient’s temperature and then to choose the proper medical treatment. This conversion of a curious toy into a measuring device and the intentional application of the data obtained for some purpose, have all the features of effective discovery [9]. This is not true for some other supposed inventors of thermometers, such as della Porta or the ‘Tausendkünstler’ Drebbel of Alkmar, who used to supply various prototypes of hydropneumatic *perpetuum mobile* to practically all of the king’s courts throughout Europe. It is quite certain, that knowledge of the thermoscope (‘weatherglass’) in the 17th century was widely spread among educated people, either due to the new edition of Heron’s papers, or the accessibility of excerpts of Arabian alchymistic manuscripts, so that to assign only one ‘true’ inventor of thermometer, from among persons such as Galilei, Segredo, Fludd, Bacon, van Helmont, Boyle and others, is practically impossible [8, 10]. Among these inventors was also Goethe, who more than one century later (1732) had patented a virginomorphic glass bowl filled with wine and provided with very strange pipe – the device was more worthy of deep psychoanalytical study than for ‘reliable forecast of weather’. However, during the second half of the 17th century there were in use advanced forms of thermometers for medical and meteorological purposes, namely those constructed by Guericke [11] and by the members of *Accademia del Cimento* in Florence who had also invented the closed fluid-thermometer. The activities of this last named institution are especially well-documented. Besides research reports, a box with original instruments was discovered in the last century by Antinori [12]. The following peripatetic (i.e. Aristotelian) explanation of a thermometer function was put forward. Coldness in the external air activates the hotness inside the bulb which then escapes most likely into the solid wall of the bulb. This process changes the ratio between the Qualities of the enclosed air, in other words its Form. The depleted Form of the air has obviously a smaller volume and the resulting empty space has to be immediately filled by water due to the *horror vacui* – nature’s abhorrence of a vacuum.

The second half of the 17th century may be characterised as an era of the differentiation of pure theoreticians and experimentalists. Typical of the theoreticians, represented e.g. by Bacon, Descartes and Gassendi, was a very prudent and critical approach to new experimental facts, a deep interest in new methodology which was more reliable than the medieval scholastics, and attempts to construct universal theories. The main positive contribution of these scientists was probably the destruction of the old theories. Very effective in this field was Bacon’s rather boring system of well-arranged tables which enabled the logical exclusion of some possible explanations of particular phenomenon. Remarkable is e.g. his conclusion that ‘...the very essence of heat is motion and nothing else’ [13]. Very ambitious was the ‘*Cosmogony*’ of Descartes. He speculated that the space of the whole Universe was filled with matter in three forms, distributed among the stars and the Sun (fire), heaven (transparent matter) and the Earth (dark matter). All observable effects in nature are then due to

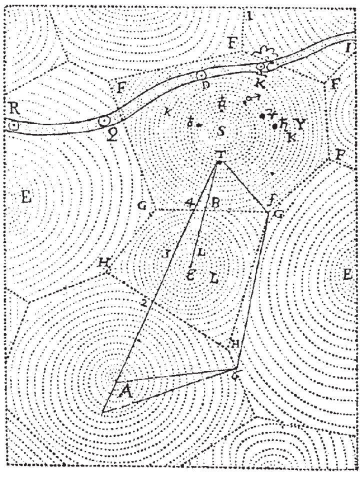

Fig. 3 Descartes’ picture of the Universe

the eternal movements of matter which forms gigantic or quite small whirls – *tourbillons* [14] (Fig. 3). Such a theory can, of course, explain everything, but unfortunately cannot predict anything. A little better was the theory proposed by Gassendi [5]. This follower of Democritos identified heat and coldness with microscopic material particles – atoms. Accordingly, the heat substance consists of spherical and very fast atoms, while the atoms of coldness are lazy tetrahedrons with sharp edges causing pain and destroying solid materials. The compatibility of this 'substantial' theory with mathematical treatments probably caused it to survive with minor changes till the 19th century. Interestingly, the premise of Bacon and Descartes that heat is a kind of motion, contradicting the opinion of Gassendi that heat consists of particles (quasi-particles) was unified in some sense by the modern kinetic theory of matter.

Regardless of progress made by the theory, it would have become sterile if not for extensive work by other experimental scientists. The words of Fludd that '...the thermometer became a mighty weapon in the Herculean fight between Truth and Falsehood ' [15] were prophetic. The most distinguished person who was trying to use this 'weapon' for quantitative measurements was Boyle. Unfortunately, the main problem of his experiments was the absence of sufficiently reproducible fixed points characterising the thermal state, so that he was able to perform only relative measurements. This serious problem was solved satisfactorily, much later, at the beginning of the 18th century by Römer and Fahrenheit [16]. They introduced fixed points to thermometry such as the freezing point of an aqueous solution of salmiac, the freezing point of water, the normal temperature of the human body and the boiling point of water. The intervals between the fixed points marked on the scale of a fluid thermometer were divided regularly into the degrees. Such a calibration, which was for some time Fahrenheit's personal secret, ensured very good reproducibility of different instruments. At the same time, an analogous method for the construction of a thermometric scale was devised independently by Amontons [10, 17] who made experiments with a constant volume gas thermometer. By extrapolating the regularly-divided (into 100 degrees) scale between the boiling and the freezing points of water bellow the freezing point, Amontons noticed that there should be a point corresponding to zero pressure of the gas in the thermometer. He called this point (lying approximately 270 degrees below the freezing point of water) the absolute zero or point of ultimate coldness (*l'extręme froid*) and suggested its use as a natural fixed point. To Amontons belongs also another fruitful idea, i.e. the use of a gas thermometer, which is not a very convenient but, nevertheless, reliable instrument, for the calibration of more practical fluid thermometers. Fahrenheit's and Amontons' scales have a lot of common features with modern thermometric scales. These enabled the fundamental problems in scientific thermometry to be solved, namely: to assign a number t, called the empirical temperature, to any given thermal state, to decide whether two bodies have the same temperature or not , and to determine which body has the higher temperature. Later Maxwell [18] recognised that for thermometry to be a logically closed system, it is necessary to add a concept of thermal equilibrium and another theorem, sometimes called the zeroth law of thermodynamics, according to which: 'Two bodies which are in thermal equilibrium with a third one are also in thermal equilibrium

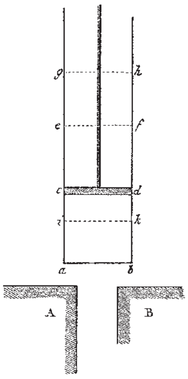

Fig. 4 Scheme of Carnot's heat engine

with each other.' By establishment of this theorem, which has the form of Euclid's first axiom, the development of the concept of empirical temperature was completed.

While investigating, theoretically, the optimisation of steam engines, Carnot devised (1824) [19] an idealised heat engine capable of performing virtual, fully computable cyclic processes. The engine consists of a thermally-insulated cylinder containing a gas and is provided with a movable piston (Fig. 4). The bottom of the cylinder can be either insulated or, in turn, put into contact with bath A or bath B which have different empirical temperatures ($t_A > t_B$). These operations may be performed in such a way that only isothermal and adiabatic changes are involved, so that it can be proved mathematically that the work done during one cycle is maximal. Using these conclusions, and the conjecture about the impossibility of *perpetuum mobile* generalised for thermal phenomena, Carnot formulated the following important theorem: 'The moving force of the fire (i.e. useful work) does not depend on the agent used for its development and its magnitude depends only on the temperatures of the bodies between which the transfer of heat takes place'. It was Kelvin's excellent idea [20] that every thermometer may be treated as a special kind of thermal engine working between a bath kept at the temperature which has to be measured and an other one at the reference temperature. According to Carnot's theorem, the efficiency (normalised useful work) of any reversible cycle is dependent only on these two temperatures, regardless of the working (thermometric) substance, so that by taking just this efficiency as a measure of the temperature, the absolute scale (i.e. independent of device and/or of material constants) can be constructed. In order to find additional conditions to which such a construction must be submitted, Kelvin made a thought experiment, with three reversible engines simultaneously working with three baths at different empirical temperatures $t_1 > t_2 > t_3$. Application of Carnot's theorem to all combinations of these cycles provided a functional equation with solution:

$$Q_1/Q_3=\varphi(t_1)/\varphi(t_3) \qquad (1)$$

where $\varphi(t)$ is a monotonic, positive, definite, real function of the empirical temperature t. The simplest choice, which later became the basis of international temperature scale (Kelvin's second suggestion [21]) is that corresponding to the relation:

$$\varphi(t)=\alpha T \tag{2}$$

where α is universal constant and $\alpha T=Q$ the heat exchanged between the bath of empirical temperature t and the thermometer. This convention is simultaneously fully consistent with the empirical scales induced by isochoric (Amonton's scale) and/or isobaric (Avogadro's scale) equations of state contained in the formula $pv/T=const.$ How important the choice of $\varphi(t)$ was for further interpretation of concepts in thermal physics will be apparent from the following example. Dalton [22], in analysing not very reliable measurements of the thermal expansion of fluids, found a quadratic dependence between supplied heat, identified by him with temperature θ, and the increase of the volume of the fluid with respect to that at its freezing point. Using this conjecture as a basis for the construction of a temperature scale, he was able to fit the isobaric equation of state of any permanent gas by the formula:

$$v/v_0=\exp(\beta(\theta-\theta_0)) \tag{3}$$

where β is a universal constant. For an ideal gas as the thermometric substance, this relation defines the temperature scale (θ) which is arbitrary, but fulfils the requirements put on the construction of an absolute scale by Kelvin. In fact, this scale is practically identical with the Kelvin's first suggestion [20]. Let us compare the law (3) with the isobaric equation of state of an ideal gas, written down using Kelvin's international scale (which is identical in this case with the scale of Avogadro).

$$v/v_0=T/T_0 \tag{4}$$

For this purpose, isobaric ($p=const.$) heating of the bath surrounding the bulb of a thermoscope (Fig. 5) will be considered. The initial state, characterised by the volume v_0 and by both temperatures θ_0 and T_0, will be changed to a new equilibrium state which corresponds to the volume v. Using Eqs (3) and (4), the values of θ and T can be determined. It is easy to show that the difference $(T-T_0)=const.\ (v-v_0)$, $(const.=pT_0/v_0)$, measured on Avogadro's scale, is directly proportional to the work done by the gas *vs.* the external pressure p, while the temperature difference on Dalton's scale measures the increase of the entropy (in its usual sense) of the gas in the thermoscope, because, from Eq. (3), $(\theta-\theta_0)=const.\ \ln(v/v_0)$, where $const.=(\beta C_v+R)$. It is remarkable that both

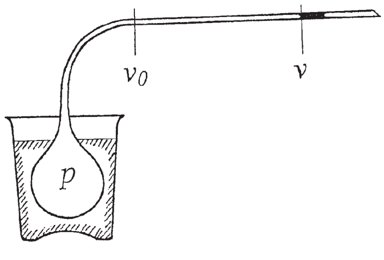

Fig. 5 An isobaric thermoscope used for the comparison of θ and T scales

the arbitrarily chosen quantities called temperature, related to just the same experimental situation (volume change induced by heating) may have such a different interpretation. There is an enormous freedom in how to choose the function, φ, but it is a very difficult task and a matter of intuition to anticipate whether such a choice will be of practical value in the future. Thus the intuitive opinion of Dalton that the temperature should reflect something closely related to the content of heat in a given body, actually corresponds to his scale. On the other hand, the simple interpretation of heat conduction and the evaluation of efficiency of, e.g., steam engines require a temperature which behaves like the potential of a heat fluid. In this case, a linear scale, equivalent to the contemporary Kelvin's international scale is the most convenient one.

The present system of classical thermodynamics is usually considered to be an example of a closed and consistent physical theory with a well-defined conceptual basis. In spite of that, thermodynamics is traditionally very difficult to learn, teach and understand. The difficulties are concentrated especially around the artificial quantity called 'entropy', which has no clear physical meaning but can always only increase. Puzzling is also the fact that the introduction of temperature as an integrating factor of a certain differential equation [3] completely eliminates heat from thermodynamics and, simultaneously, temperature as an intensive quantity which pretends to be a measure of the potential of heat in particular problems. Another source of the difficulties is the necessity of distinguishing between reversible (non-existing) and irreversible (natural) processes, the former being convenient for theoretical treatments, while the latter are good for applications. These circumstances would convert thermodynamics into an esoteric doctrine, if it were not for the fact that the mathematics behind it works very satisfactorily and that the thermodynamic predictions are essentially correct.

This poor understanding of thermodynamics and its incompatibility with common sense are due to the very unhappy choice of the conceptual basis used for the description of thermal processes in the 19th century, as was pointed out by Mach [23] and by Job [24].

What are then the main flaws in classical thermodynamics? At the end of the 18th century Count Rumford [25] discovered that mechanical work can be converted into heat without limitations, so that heat is likely not to be a substance, as Black and Carnot thought, but is rather a kind of motion. This view was confirmed by theoretical considerations of Mayer [26] and especially by the extensive experiments made by Joule [27], who found that the heat developed in a dissipation process is always proportional to the work done and that the proportionality factor is universal, i.e. independent of the particular process. Generalisation of these facts led to the establishment of the principle of equivalence of heat and mechanical work [28]. This identification of heat with some form of energy (thermal energy) led to the first law of thermodynamics, i.e. the energy conservation law, which included heat as a special additive term. Unfortunately, this identification of heat with some form of energy (thermal energy) is not correct. There are a lot of processes in which mechanical work is fully converted into heat, but no single process enabling complete conversion of a given amount of heat back into mechanical work without other changes. The very absence of this reverse transformation excludes logically the possibility that heat is equivalent to any kind of energy, for which the unlimited exchanges among its particular forms are characteristic. This serious inconsistency in classical thermodynamics is compensated by the introduction of a new quantity called entropy and of a new axiom, the second law of thermodynamics, which states that the entropy never decreases ('is indestructible') and increases ('is created') during every irreversible process. No wonder that a meaningful physical interpretation of this quantity is lacking. This is due to the fact that it has just the same properties as heat (in its common sense), the name of which has already been quite improperly used to label a certain special kind of energy. Indeed, if we identify heat with entropy, the mysterious second law of thermodynamics becomes quite intuitive and very easy to understand (cf. 'Heat cannot be annihilated in any real physical process'). For instance, in an experiment where heat (=entropy) is generated by means of friction of two blocks of any material, it is clear at first glance that the heat (=entropy) will not disappear by moving the blocks in opposite direction but it will further increase. As concerns the reversibility of such a process, it can be approached only by effectively decreasing the friction, i.e. by suppressing the generation of heat (=entropy) during the movement and, eventually, there is no concept of what kind of movements should be done with the said blocks to completely destroy the already developed heat (=entropy). Moreover, the substitution of the word 'entropy' by the word 'heat', which is no longer regarded as a kind of energy, enables the intelligible interpretation of temperature as a potential of heat in closer analogy with other potentials (electric, gravitational) used in other branches of physics.

In conclusion, a brief outline of the history of thermal physics has been given and some persisting imperfections in the conceptual basis of classical thermodynamics have been pointed out. The removal of these flaws, by careful revision, would, I believe, be advantageous for the further development of thermal physics, in spite of the horrifying extent of the required changes which would very likely, be followed by an enormous impact on science and common life.

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